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SYSTEMATIC RESUMMED PERTURBATION THEORY

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Abstract

A systematic loop expansion is formulated in terms of full propagators and vertices. It is based on an expansion of the general solution of an exact non-perturbative flow equation.

One-particle irreducible vertices are the basic building blocks for the computation of cross sections and decay rates in particle physics or for the critical equation of state in statistical physics. They are often computed in a perturbative or loop expansion as functions of renormalized coupling constants. In some circumstances one would like to use instead of constant couplings the full momentum-dependent vertices and propagators in the loop integrals. As an example we mention the scale ambiguity: The renormalized couplings used in perturbation theory are defined by n-point functions evaluated at a given scale μ for the momenta. Renormalization group equations describe the scale dependence of the corresponding renormalized coupling $g(\mu)$. A one-loop calculation for some different n-point function corresponds to a certain power of $g(\mu)$ and one always encounters a scale ambiguity: At what scale should the coupling be taken? The difference between the choice of two distinct scales is of higher loop order and cannot be settled within the one-loop calculation. On the other hand, one often has knowledge about the momentum dependence of the full vertex by which $g(\mu)$ is defined, and one may be tempted to use this for a more accurate determination already of the one-loop graph. If one could replace $g(\mu)$ in the loop integral by the full momentum-dependent vertex, the scale ambiguity would disappear. There would be no need to select a particular renormalization scale μ since one integrates over the momenta appearing in the vertex. The inherent difficulty in such an approach concerns the compatibility with higher loop orders. Since the use of momentum-dependent vertices corresponds to a resummation of higher orders of $g(\mu)$, one must avoid a double counting of these effects once the two-loop correction is included. The question arises of how to formulate a systematic expansion where only full vertices and propagators instead of the (renormalized) classical ones appear in every order of the expansion. Applications of such an expansion would be much wider than the mentioned scale ambiguity problem. We only mention here the use of solutions of gap equations in loop integrals for other n-point functions. There appears again a double counting problem in two-loop order which has to be treated systematically.

We propose in this letter a way of avoiding such double counting by developing a systematic resummed perturbation theory formulated directly in terms of renormalized n-point functions. It is based on the formal solution of an exact flow equation which describes the change of the effective action Γ_k with the infrared cutoff scale

k in terms of a renormalization-group improved one-loop expression [1]. The exact flow equation is closely related to the Wilsonian approach to the renormalization group equations [2]. At the end of this letter we will sketch several possible applications of systematic resummed perturbation theory (SRPT) which go beyond the range of problems accessible within standard perturbation theory.

We start from the exact non-perturbative flow equation[1] ($t = \ln \frac{k}{\Lambda}$)

$$\partial_t \Gamma_k[\varphi] = \frac{1}{2}\, Tr\, \left\{\partial_t \mathcal{R}_k\, (\Gamma_k^{(2)}[\varphi] + \mathcal{R}_k)^{-1}\right\} \tag{1}$$

It is obtained by adding to the classical action a quadratic infrared cutoff

$$\Delta_k S = \frac{1}{2}\, Tr\, \{\mathcal{R}_k\, (\bar{\chi} \otimes \chi)\} \tag{2}$$

and differentiating the functional integral which defines the k-dependent effective action Γ_k with respect to k [1]. Here the trace includes a summation over internal indices as well as a momentum integration, $Tr = \int \frac{d^d q}{(2\pi)^d} \sum_a$, and χ stands for the fluctuations $\chi_a(q)$ over which the functional integral is performed. The flow equation (1) involves the exact field-dependent inverse propagator

$$(\Gamma_k^{(2)})_{ab}(q, q') = \frac{\delta^2 \Gamma_k}{\delta \varphi_a(q) \delta \bar{\varphi}_b(q')} \tag{3}$$

(We use here a language with matrices $\Gamma_k^{(2)}, \mathcal{R}_k$ and $(\bar{\chi} \otimes \chi)_{ba}(q', q) = \bar{\chi}_b(q')\chi_a(q)$.) The infrared cutoff $\mathcal{R}_k$ is typically (but not necessarily) diagonal in momentum space and should vanish for $k \to 0$. An example is

$$(\mathcal{R}_k)_{ab}(q, q') = Z_k q^2 (e^{q^2/k^2} - 1)^{-1} (2\pi)^d \delta(q - q') \delta_{ab} \tag{4}$$

The "average action" Γ_k is a functional of "classical" fields φ. For $k \to 0$ it becomes the usual effective action, i.e. the generating functional for the 1PI Green functions. Our aim is a suitable loop expansion of the general solution of the flow equation (1) for $\Gamma[\varphi] = \Gamma_{k \to 0}[\varphi]$. Indeed, we may use the flow equation for an implicit definition of the theory, with all short-distance physics and regularization encoded in the "initial value" $\Gamma_\Lambda[\varphi]$ specified at some high momentum scale Λ.

Standard perturbation theory can easily be recoverd from an iterative solution of the flow equation (1). Starting from the leading or "classical" contribution $\Gamma_{k(0)} \equiv$

[1] We restrict the discussion in this letter to bosons. The generalization to fermions is straightforward, with a supertrace in (1).

Γ_Λ one may insert this instead of Γ_k on the r.h.s. of (1). Performing the t-integration generates the one-loop contribution

$$\Gamma_k - \Gamma_\Lambda = \frac{1}{2} Tr \left\{ \ln(\Gamma_\Lambda^{(2)} + \mathcal{R}_k) - \ln(\Gamma_\Lambda^{(2)} + \mathcal{R}_\Lambda) \right\}, \tag{5}$$

where we remind that $\mathcal{R}_k \to 0$ for $k \to 0$. We observe that the momentum integration on the r.h.s. of (5) is regularized in the ultraviolet through subtraction of $\ln(\Gamma_\Lambda^{(2)} + \mathcal{R}_\Lambda)$. This is a type of implicit Pauli-Villars regularization with the heavy mass term replaced by a momentum-dependent piece $\mathcal{R}_\Lambda$ in the inverse propagator. With suitable chirally invariant $\mathcal{R}_\Lambda$ [3] this can be used for a regularization of models with chiral fermions. Also gauge theories can be regularized in this way, but care is needed since Γ_Λ has to obey identities reflecting the gauge invariance [4]. Going further, the two-loop contribution obtains by inserting the one-loop expression for $\Gamma_k^{(2)}$ as obtained from eq. (5) into the r.h.s. of (1). Only the classical inverse propagator $\Gamma_\Lambda^{(2)}$ and its functional derivatives appear in the nested expressions. They are independent of k and the integration of the approximated flow equation is straightforward.

It is the purpose of this letter to replace the perturbative iteration sketched above by a new one which involves the full propagators and vertices instead of the classical ones. We start again with the lowest order term

$$\Gamma_{k(0)}[\varphi] = \Gamma_\Lambda[\varphi] \tag{6}$$

where Λ is now some conveniently chosen scale (not necessarily the ultraviolet cutoff). In the next step we write equation (1) in the form

$$\partial_t \Gamma_k = \frac{1}{2}\, Tr\; \partial_t \ln(\Gamma_k^{(2)} + \mathcal{R}_k) - \frac{1}{2}\, Tr\; \left\{ \partial_t \Gamma_k^{(2)} (\Gamma_k^{(2)} + \mathcal{R}_k)^{-1} \right\} \tag{7}$$

Here $\partial_t \Gamma_k^{(2)}$ can be inferred by taking the second functional derivative of eq. (1) with respect to the fields φ. Equation (7) can be taken as the starting point of a systematic loop expansion by counting any t-derivative acting only on Γ_k or its functional derivatives as an additional order in the number of loops. From (1) it is obvious that any such derivative involves indeed a new momentum loop. It will become clear below that in case of weak interactions it also involves a higher power in the coupling constants. The (modified) one-loop contribution $\Gamma'_{k(1)}$ can now be defined by

$$\Gamma_k = \Gamma_{k(0)} + \Gamma'_{k(1)} + \Gamma_{k(1)}^{(R)} \tag{8}$$

with

$$\Gamma'_{k(1)} = \frac{1}{2}\, Tr\, \left\{ \ln(\Gamma_k^{(2)} + \mathcal{R}_k) - \ln(\Gamma_k^{(2)} + \mathcal{R}_\Lambda) \right\} \tag{9}$$

In contrast to eq. (5) the r.h.s. involves now the full (field-dependent) inverse propagator $\Gamma_k^{(2)}$, and, by performing suitable functional derivatives, the full proper vertices. Putting $k = 0$ the resummed one-loop expression (9) ressembles a Schwinger-Dyson [5] or gap equation, but in contrast to those only full vertices appear. For example, for $k = 0$ the one-loop contribution to the inverse propagator obtains by taking the second functional derivative of eq. (9)

$$\begin{aligned} \left(\Gamma'^{(2)}_{(1)}\right)_{ab}(q,q') &= \frac{1}{2}\, Tr \left\{ \left(\Gamma^{(2)}\right)^{-1} \frac{\delta^2 \Gamma^{(2)}}{\delta\varphi_a(q)\delta\bar{\varphi}_b(q')} \right\} \\ &\quad -\frac{1}{2}\, Tr\, \left\{ \left(\Gamma^{(2)}\right)^{-1} \frac{\delta\Gamma^{(2)}}{\delta\varphi_a(q)} \left(\Gamma^{(2)}\right)^{-1} \frac{\delta\Gamma^{(2)}}{\delta\bar{\varphi}_b(q')} \right\} \\ &\quad -\text{regulator terms} \end{aligned} \tag{10}$$

and involves the proper three- and four-point vertices. Adding the lowest order piece $\Gamma^{(2)}_{(0)}$ and approximating the vertices by their lowest order expressions, eq. (10) reduces to the standard gap equation for the propagator in a regularized form. Assuming that this is solved (for example numerically by an iterative procedure) we see that the resummed one-loop expression (10) involves already arbitrarily high powers in the coupling constant, and, in particular, contains part of the perturbative two-loop contribution. The remaining part of the perturbative two-loop contribution must appear in the resummed two-loop contribution that we will discuss next.

The remaining piece beyond resummed one-loop order $\Gamma^{(R)}_{k(1)}$ obeys the flow equation

$$\begin{aligned} \partial_t \Gamma^{(R)}_{k(1)} &= -\frac{1}{2}\, Tr\, \left\{ \partial_t \Gamma_k^{(2)} \left[\left(\Gamma_k^{(2)} + \mathcal{R}_k\right)^{-1} - \left(\Gamma_k^{(2)} + \mathcal{R}_\Lambda\right)^{-1} \right] \right\} \\ &= \frac{1}{4} \left[\left(\Gamma_k^{(2)} + \mathcal{R}_k\right)^{-1} - \left(\Gamma_k^{(2)} + \mathcal{R}_\Lambda\right)^{-1} \right]_{\sigma\tau} \frac{\delta^4 \Gamma_k}{\delta\varphi_\tau \delta\bar{\varphi}_\sigma \delta\varphi_\alpha \delta\bar{\varphi}_\beta} (\Gamma_k^{(2)} + \mathcal{R}_k)^{-1}_{\beta\gamma} \\ &\quad (\partial_t \mathcal{R}_k)_{\gamma\delta} (\Gamma_k^{(2)} + \mathcal{R}_k)^{-1}_{\delta\alpha} \\ &\quad -\frac{1}{2} \left[\left(\Gamma_k^{(2)} + \mathcal{R}_k\right)^{-1} - \left(\Gamma_k^{(2)} + \mathcal{R}_\Lambda\right)^{-1} \right]_{\sigma\tau} \frac{\delta^3 \Gamma_k}{\delta\varphi_\tau \delta\varphi_\alpha \delta\bar{\varphi}_\beta} \left(\Gamma_k^{(2)} + \mathcal{R}_k\right)^{-1}_{\beta\gamma} \\ &\quad \frac{\delta^3 \Gamma_k}{\delta\bar{\varphi}_\sigma \delta\varphi_\gamma \delta\bar{\varphi}_\delta} \left(\Gamma_k^{(2)} + \mathcal{R}_k\right)^{-1}_{\delta\epsilon} (\partial_t \mathcal{R}_k)_{\epsilon\eta} \left(\Gamma_k^{(2)} + \mathcal{R}_k\right)^{-1}_{\eta\alpha} \end{aligned} \tag{11}$$

where $\alpha, \beta, ...$ etc combine internal indices and momentum labels. Using the sym-

metries of (11) it is straightforward to extract the resummed two-loop contribution

$$\Gamma^{(R)}_{k(1)} = \Gamma'_{k(2)} + \Gamma^{(R)}_{k(2)} \tag{12}$$

by writing (11) as a total t-derivative plus terms where ∂_t acts only on Γ_k and its functional derivatives. One finds

$$\begin{aligned}\Gamma'_{k(2)} &= -\frac{1}{8}\,\frac{\delta^4\Gamma_k}{\delta\varphi_\tau\delta\bar\varphi_\sigma\delta\varphi_\alpha\delta\bar\varphi_\beta}G_{\sigma\tau}G_{\beta\alpha} \\ &\quad +\frac{1}{6}\,\frac{\delta^3\Gamma_k}{\delta\varphi_\tau\delta\varphi_\alpha\delta\bar\varphi_\beta}\,\frac{\delta^3\Gamma_k}{\delta\bar\varphi_\sigma\delta\varphi_\gamma\delta\bar\varphi_\delta}\,G_{\sigma\tau}G_{\beta\gamma}\left(G_{\delta\alpha}+\frac{3}{2}\left(\Gamma^{(2)}_k+\mathcal{R}_\Lambda\right)^{-1}_{\delta\alpha}\right)\end{aligned} \tag{13}$$

with regularized propagator

$$G_{\sigma\tau} = \left(\Gamma^{(2)}_k+\mathcal{R}_k\right)^{-1}_{\sigma\tau} - \left(\Gamma^{(2)}_k+\mathcal{R}_\Lambda\right)^{-1}_{\sigma\tau} \tag{14}$$

At this point it becomes obvious how the iterative construction of higher resummed loop terms proceeds: The flow equation for $\Gamma^{(R)}_{k(2)}$ can be written in the form

$$\partial_t\Gamma^R_{k(2)} = (\tilde\partial_t - \partial_t)\Gamma'_{k(2)} \tag{15}$$

with $\tilde\partial_t$ acting only on $\mathcal{R}_k$ $\left(\tilde\partial_t = (\partial_t\mathcal{R}_k)\frac{\partial}{\partial\mathcal{R}_k},\ \tilde\partial_t\Gamma'_{k(2)} = \partial_t\Gamma^{(R)}_{k(1)}\right)$. It involves t-derivatives acting on Γ_k and its functional derivatives with respect to the fields. Using (1), they can be expressed in terms of $\partial_t\mathcal{R}_k$ in one higher loop order. The resummed three-loop contribution obeys now $\tilde\partial_t\Gamma'_{k(3)} = \partial_t\Gamma^{(R)}_{k(2)}$ and so on.

Comparing the resummed two-loop contribution (13) with the standard two-loop contribution, we find the same type of graphs (cf. fig. 1). Only the weight factors have changed - they are now $-\frac{1}{8}$ and $\frac{1}{6}$ instead of $\frac{1}{8}$ and $-\frac{1}{12}$ in standard perturbation theory. And, of course, the graphs involve now full propagators and vertices instead of the classical ones. The difference in the weight factors is easily understood: The resummed one-loop contribution $\Gamma'_{(1)}$ also contains terms with the same structure as the two-loop contribution, once we expand eq. (9) iteratively. In fact, the functional form of Γ_k is not known exactly and one has to use a truncation $\Gamma^{(tr)}_k$ on the r.h.s. of eqs. (9) and (13). We expand $\Gamma'_{k(1)}$ in powers of the difference $D_k = \Gamma_k - \Gamma^{(tr)}_k$

$$\Gamma'_{k(1)} = \Gamma_{k(1)} + \Delta\Gamma_{k(2)} + ... \tag{16}$$

with truncated one-loop contribution

$$\Gamma_{(1)} = \frac{1}{2}\,\mathrm{Tr}\ln\Gamma^{(tr)(2)} \tag{17}$$

(We use here $k = 0$ and omit the regulator terms.) The second piece $\Delta\Gamma_{k(2)} = \frac{1}{2}\, Tr\, G^{(tr)} D^{(2)}_{k(1)}$ obtains by inserting the lowest order for D_k

$$\begin{aligned} D_{k(1)} &= \frac{1}{2} Tr \left\{ \ln\left(\Gamma_k^{(tr)(2)} + \mathcal{R}_k\right) - \ln\left(\Gamma_k^{(tr)(2)} + \mathcal{R}_\Lambda\right)\right\} \\ &\quad - \left(\Gamma_k^{(tr)} - \Gamma_\Lambda\right) \end{aligned} \tag{18}$$

and is counted as a two-loop contribution

$$\begin{aligned} \Delta\Gamma_{k(2)} &= \Delta\Gamma^{(ct)}_{k(2)} + \frac{1}{4} \frac{\delta^4 \Gamma_k^{(tr)}}{\delta\varphi_\tau \delta\bar{\varphi}_\sigma \delta\varphi_\alpha \delta\bar{\varphi}_\beta} G^{(tr)}_{\sigma\tau} G^{(tr)}_{\beta\alpha} \\ &\quad - \frac{1}{4} \frac{\delta^3 \Gamma_k^{(tr)}}{\delta\varphi_\tau \delta\varphi_\alpha \delta\bar{\varphi}_\beta} \; \frac{\delta^3 \Gamma_k^{(tr)}}{\delta\bar{\varphi}_\sigma \delta\varphi_\gamma \delta\bar{\varphi}_\delta} G^{(tr)}_{\sigma\tau} G^{(tr)}_{\beta\gamma} \left(G^{(tr)}_{\delta\alpha} + 2 \left(\Gamma_k^{(tr)(2)} + \mathcal{R}_\Lambda\right)^{-1}_{\delta\alpha}\right) \end{aligned} \tag{19}$$

$$\Delta\Gamma^{(ct)}_{k(2)} = \frac{1}{2}\, Tr \left\{ G^{(tr)} \left(\Gamma^{(2)}_\Lambda - \Gamma^{(tr)(2)}_k\right)\right\} \tag{20}$$

We notice that $\Delta\Gamma^{(ct)}_{k(2)}$ has the structure of a counter term insertion in a one-loop integral whereas the last two terms in eq. (19) correspond to the graphs in fig. 1 with weight factors $\frac{1}{4}$ and $-\frac{1}{4}$. Combining $\Delta\Gamma_{(2)}$ with the lowest order of $\Gamma'_{(2)}$ (13) one finally obtains the truncated two-loop contribution ($k = 0$ and omitting regulator terms)

$$\begin{aligned} \Gamma_{(2)} &= \frac{1}{8} \left(\Gamma^{(tr)(2)}\right)^{-1}_{\sigma\tau} \frac{\delta^4 \Gamma^{(tr)}}{\delta\varphi_\tau \delta\bar{\varphi}_\sigma \delta\varphi_\alpha \delta\bar{\varphi}_\beta} \left(\Gamma^{(tr)(2)}\right)^{-1}_{\beta\alpha} \\ &\quad - \frac{1}{12} \left(\Gamma^{(tr)(2)}\right)^{-1}_{\sigma\tau} \frac{\delta^3 \Gamma^{(tr)}}{\delta\varphi_\tau \delta\varphi_\alpha \delta\bar{\varphi}_\beta} \left(\Gamma^{(tr)(2)}\right)^{-1}_{\beta\gamma} \frac{\delta^3 \Gamma^{(tr)}}{\delta\bar{\varphi}_\sigma \delta\varphi_\gamma \delta\bar{\varphi}_\delta} \left(\Gamma^{(tr)(2)}\right)^{-1}_{\delta\alpha} + \Delta\Gamma^{(ct)}_{(2)} \end{aligned} \tag{21}$$

For the simplest ansatz $\Gamma^{(tr)} = \Gamma_\Lambda$ one recovers standard unrenormalized perturbation theory. We note that renormalized perturbation theory in a given scheme can also be recovered directly in our language as a special case: It corresponds to the truncation where $\Gamma^{(tr)}$ consists only of the n-point functions appearing in the classical action, with coefficients given by renormalized couplings as determined by the scheme-dependent renormalization conditions. Consider the example of the φ^4-theory with renormalized quartic coupling $\lambda(\mu)$ defined by the four-point function at symmetric momenta. In this scheme the contribution to the four-point function contained in D is $\sim \Delta\lambda_4(q_1, q_2, q_3, q_4)\varphi(q_1)\varphi(q_2)\varphi(q_3)\varphi(q_4)$ with $\Delta\lambda_4 = 0$ for symmetric momenta with scale μ. The scheme and scale dependence of the perturbative results appears in this language through the dependence of D on the renormalization

condition and μ. In close analogy, the one-loop result of truncated SRPT will always exhibit a "truncation dependence", but not necessarily the usual scale dependence.

Systematic resummed perturbation theory is particularly convenient for a computation of ultraviolet finite n-point functions (as the φ^6 coupling) or differences of n-point functions at different momenta (e.g. $\Delta\lambda_4$). For $\Lambda \to \infty$ all dependence on the effective ultraviolet cutoff Λ is absorbed in the renormalized couplings. In this limit the regulator terms vanish. This is very useful in the case of QCD or other gauge theories where calculations with an explicit regulator function $\mathcal{R}_\Lambda$ are quite cumbersome[2]. (Of course, ultraviolet finiteness requires here that appropriate Slavnov-Taylor or background field identities [4] are respected by $\Gamma^{(tr)}$.) We note that all Green functions can be made ultraviolet finite by appropriate subtractions. For the two-point function a typical infrared-finite quantity is $\lambda_2(q) - \lambda_2(0) - \frac{\partial\lambda_2}{\partial q^2}_{|q^2=\mu^2} q^2$ with $\Gamma^{(2)} = \lambda_2(q)(2\pi)^d\delta(q-q')$ for constant vacuum fields.

The necessary cancellation of the cutoff(Λ)-dependence for ultraviolet finite quantities has some subtleties once the truncation goes beyond renormalized perturbation theory. In this case a given loop order in SRPT involves for the higher n-point functions also diagrams not present in standard perturbation theory. For example, the one-loop order for the four-point function obtains by suitable functional differentiation of eq. (9) or (17) and involves up to six-point functions.[3] We have depicted in fig. 2 the contribution from the φ^6 coupling in a typical scalar theory. In the limit $\Lambda \to \infty$ one encounters not only the usual ultraviolet divergences. Higher order couplings present in $\Gamma^{(tr)}$ and absent in the classical action can induce additional divergences, as for example a possible divergence $\sim \Lambda^2$ from the graph of fig. 2 for the four-dimensional scalar theory. This divergence is an artefact of the expansion and cancelled by two-loop contributions of the type depicted in fig. 3. The exact definition of truncated SRPT and therefore the structure of divergences in a given loop order is specified by the ansatz for $\Gamma_k^{(tr)}$.[4] A good guide is certainly to avoid ultraviolet divergences in one-loop order for quantities which must be ultraviolet finite. This may require a tuning of the "higher order couplings" (e.g. the φ^6 cou-

[2]One can also combine SRPT with dimensional regularization such that the limit $\Lambda \to \infty$ can be taken even for UV-divergent quantities.

[3]The relation to standard perturbation theory follows from an iterative solution for the higher n-point functions.

[4]The optimum choice can be adapted to the quantity one wants to compute.

pling λ_6 or $\Delta\lambda_4$) used in the ansatz for $\Gamma^{(tr)}$. Such a tuning is no accident since the values of the higher order couplings in a given model are fixed by an infrared fixed-point behaviour. They are no free parameters but rather computable in terms of the renormalized couplings. The requirement of vanishing ultraviolet divergences can be used as a constraint for the higher order couplings taken into account in $\Gamma_k^{(tr)}$.

The aim of the proposed systematic resummed perturbation theory is not only the computation of Green functions in a given order in the coupling constant. For this purpose standard perturbation theory is sometimes more direct and efficient. The systematic character of SRPT guarantees that in every loop order the standard perturbative contributions are all included. Beyond that, however, a given loop order in SRPT also contains partial higher-loop effects of standard perturbation theory. The convergence of SRPT is not necessarily related to the existence of a small coupling constant. From eq. (7) we learn that the total size of all higher-loop contributions as compared to the one-loop contribution is controled by the relative size of the piece arising from $\partial_t\Gamma_k^{(2)}$ as compared to $\partial_t(\Gamma_k^{(2)}+R_k)$. In particular, if Γ_k changes only slowly with k - for example in case of an effective physical IR-cutoff - one expects SRPT to converge rather well.

The virtues of SRPT particularly pertain to situations at the borderline of validity of standard perturbation theory and the possible combination with non-perturbative approaches. Among the possible useful applications we mention the combination of SRPT with gap equations for the mass term. Gap equations are frequently used for situations where the renormalized mass terms differ substantially from the bare mass term (even after subtraction of counterterms), as, for example, in case of spontaneous symmetry breaking or high temperature field theory. SRPT allows for the use of solutions of gap equations within a systematic loop expansion. This becomes possible since $\Gamma_k^{(tr)(2)}$ instead of $\Gamma_\Lambda^{(2)}$ appears systematically in all graphs. In addition, SRPT may also be used as a starting point for the gap equation which determines the mass term in $\Gamma^{(tr)}$ (cf. eq. (10)). It has the advantage that no short-distance couplings (bare couplings) enter this equation. (The latter contrasts with the Schwinger-Dyson equations which are often used as a starting point for gap equations.)

A second issue concerns the possibility of a systematic split of loop graphs into high and low momentum contributions. This may be advantageous when the physics

of the low momentum modes differs qualitatively from the one for the high momentum modes, as in QCD. In fact, the successful QCD sum rules are employed in this spirit, even though not fully systematic so far. A given n-loop contribution of SRPT can always be split

$$\Gamma_{(n)} = \Gamma_{k_c(n)} + \left(\Gamma_{(n)} - \Gamma_{k_c(n)}\right) \tag{22}$$

where $\Gamma_{k_c(n)}$ contains an explicit IR-cutoff $\mathcal{R}_{k_c}$ and only fluctuations with momenta $q^2 \gtrsim k_c^2$ are effectively included in this piece. The second piece, $\Gamma_{(n)} - \Gamma_{k_c(n)}$, includes the remaining effects of modes with $q^2 \lesssim k_c^2$. For this piece k_c acts as an effective ultraviolet cutoff which guarantees that only the low momentum modes are counted. For a demonstration, let us consider the one-loop contribution (9) with $\Lambda \equiv k_c$, $k = 0$. This assumes that one has already computed the "perturbative part" Γ_{k_c} which acts now as the classical part $\Gamma_{(0)} = \Gamma_\Lambda = \Gamma_{k_c} = \Gamma^{(tr)}$ for the remaining calculation. The "non-perturbative part"

$$\Gamma^{(np)}_{(1)} \equiv \Gamma_{(1)} - \Gamma_{k_c(1)} = \frac{1}{2}\, Tr\left\{\ln \Gamma^{(2)} - \ln\left(\Gamma^{(2)} + \mathcal{R}_{k_c}\right)\right\} \tag{23}$$

encodes the infrared physics and exhibits the UV-cutoff k_c. Functional derivatives of this quantity (for example with respect to heavy quark fields in a QCD calculation) lead to the appearance of momentum integrals over propagators (for example for gluons). These momentum integrals are UV-regularized and can be associated with expectation values of corresponding regularized composite operators. In a QCD computation this involves regularized operators for gluon and quark condensates. which can be parametrized phenomenologically. For a systematic formulation of these concepts it is obviously crucial that the full propagator $\left(\Gamma^{(2)}\right)^{-1}$ and not the classical propagator appears in the relevant momentum integrals.

Finally, SRPT could be used in connection with approximative solutions of the flow equation (1) in the following way: Eq. (1) is a functional differential equation which cannot be reduced to a closed system for a finite number of couplings. For example, the beta function for the four-point vertex (the fourth functional derivative of the r.h.s. of eq. (1)) involves not only two-, three- and four-point functions, but also up to six-point functions. Approximate solutions to the flow equation often proceed by truncation. For example, contributions involving the five- and six-point function could be neglected. As an alternative, these higher n-point functions could be evaluated by SRPT. (Closely related ideas have already been tested successfully for

four-dimensional scalar theories [6].) We observe that the momentum integrals relevant for the higher n-point functions (also for differences of lower n-point functions at different momenta) are usually dominated by the low momentum modes with $q^2 \approx k^2$. This motivates the use of SRPT rather than standard perturbation theory for this purpose. Conversely, the use of (approximated) full propagators and vertices as determined by the solution of renormalization group or flow equations may be advantageous for the estimate of some dominant infrared effects (renormalons etc.), which would only appear in higher order in standard perturbation theory.

In summary, we have developed a systematic resummed perturbation theory which is formulated in terms of full propagators and vertices (eqs. (9), (13)) or approximations thereof (eqs. (17),(21)). The truncated version ressembles very closely standard perturbation theory with classical action replaced by the truncated effective action $\Gamma^{(tr)}$. Although we have only sketched here some possible applications, we hope that we have convinced some of the readers of this letter that SRPT can be useful in practice for certain types of problems in particle physics and statistical physics where standard perturbative results become problematic.

References

[1] C. Wetterich, Phys. Lett. **B301** (1993) 90;
M. Bonini, M. D'Attanasio, and G. Marchesini, Nucl. Phys. **B409** (1993) 441

[2] F. Wegner, A.Houghton, Phys. Rev. **A8** (1973) 401;
K. G. Wilson, I. G. Kogut, Phys. Rep. **12** (1974) 75;
S. Weinberg, *Cirtical Phenomena for Field Theorists*, Erice Subnucl. Phys. (1976) 1;
J. Polchinski, Nucl. Phys. **B231** (1984) 269;
A. Hasenfratz, P. Hasenfratz, Nucl. Phys. **B270** (1986) 685

[3] C. Wetterich, Z. Phys. **C48** (1990) 693;
D. Jungnickel and C. Wetterich, preprint HD-THEP-95-7

[4] M. Reuter and C. Wetterich, Nucl. Phys. **B417** (1994) 181, **B427** (1994) 291, preprint HD-THEP-94-39;
C. Becchi, *On the construction of renormalized quantum field theory using*

renormalization group techniques, in: Elementary Particles, Field Theory and Statistical Mechanics, eds. M. Bonini, G. Marchesini, and E. Onofri, Parma University, 1993;
M. Bonini, M. D'Attanasio, and G. Marchesini, Nucl. Phys. **B418** (1994) 81; **B421** (1994) 429; **B437** (1995) 163; Phys. Lett. **B346** (1995) 87;
U. Ellwanger, Phys. Lett. **B335** (1994) 364;
U. Ellwanger, M. Hirsch, and A. Weber, preprint LPTHE Orsay 95-39, to appear in Z. Phys. C:
F. Freire and C. Wetterich, preprint HD-THEP-96-1

[5] F. J. Dyson, Phys. Rev. **75** (1949) 1736;
J. Schwinger, Proc. Nat. Acad. Sc. **37** (1951) 452, 455

[6] T. Papenbrock and C. Wetterich, Z. Phys. **C65** (1995) 519

Figure captions

Fig. 1: Graphs contributing in two-loop order.

Fig. 2: One-loop contribution to the four-point function from a six-point vertex.

Fig. 3: The two-loop contributions to the four-point function resulting from contractions of the effective six-point function in one-loop order.